Sequences and Series

Question1

The value of the greatest positive integer k, such that 49^k+1 is a factor of $48\left(49^{125}+49^{124}+\ldots+49^2+49+1\right)$ is

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Options:

A.

32

B.

63

C.

65

D.

60

Answer: B

Solution:

We have, 49^k+1 is a factor of

$48\left(49^{125}+49^{124}+\ldots+49^2+49+1\right)$

Let $S=49^{125}+49^{124}+\ldots+49+1$

$$\Rightarrow \quad S=\frac{49^{126}-1}{49-1}=\frac{49^{126}-1}{48} \Rightarrow 48 S=49^{126}-1$$

Since, 49^k+1 divides $49^{126}-1$

$\therefore k$ divides 126 .

$\Rightarrow \quad 2k$ divides 126

$\Rightarrow \quad$ Greatest possible integer $k = 63$

Question2

$1 + (1 + 3) + (1 + 3 + 5) + (1 + 3 + 5 + 7) + \ldots$ **to 10 terms** $=$

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Options:

A.

385

B.

285

C.

506

D.

406

Answer: A

Solution:

Given series

$1 + (1 + 3) + (1 + 3 + 5) + (1 + 3 + 5 + 7) + \ldots$

to 10 terms.

The nth term of the series is the sum of first n odd numbers.

$$
\begin{aligned}
&So,\ T_n = 1 + 3 + 5 + \ldots + (2n-1) = n^2 \\
\therefore\quad & T_1 = 1^2 = 1 \\
& T_2 = 2^2 = 4 \\
& T_3 = 3^2 = 9 \\
& \quad\vdots \\
& T_{10} = 10^2 = 100
\end{aligned}
$$

Thus, $S_{10} = \sum_{n=1}^{10} n^2$

We know that the sum of the first k squares is

$\sum_{n=1}^{k} n^2 = \frac{k(k+1)(2k+1)}{6}$

For $k = 10$

$$
\begin{aligned}
S_{10} &= \frac{10(10+1)(2 \times 10+1)}{6} \\
&= \frac{10 \times 11 \times 21}{6} = \frac{2310}{6} = 385
\end{aligned}
$$

$\therefore S_{10} = 385$

Question3

If $1 \cdot 3 \cdot 5 + 3 \cdot 5 \cdot 7 + 5 \cdot 7 \cdot 9 \ldots$ to n terms $= n(n+1)f(n)$, then $f(2) =$

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Options:

A. 12

B. 42

C. 18

D. 20

Answer: D

Solution:

$\mathrm{t}_n = (2n-1)(2n+1)(2n+3)$

$t_n = 8n^3 + 12n^2 - 2n - 3$

$$S_n = \Sigma S_n = 8\Sigma n^3 + 12\Sigma n^2 - 2\Sigma n - \Sigma 3$$

$$= 8\left(\frac{n(n+1)}{2}\right)^2 + 12\frac{n(n+1)(2n+1)}{6} - 2\frac{n(n+1)}{2} - 3n$$

$$= n\left[\frac{8 \cdot n(n+1)^2}{4} + 2(n+1)(2n+1) - n - 1 - 3\right]$$

$$= n\left[2n\left(n^2+1+2n\right) + 2\left(2n^2+3n+1\right) - n - 4\right]$$

$$= n\left[2n^3 + 4n^2 + 2n + 4n^2 + 6n + 2 - n - 4\right]$$

$$= n\left[2n^3 + 8n^2 + 7n - 2\right]$$

$$= n(n+1)f(n)$$

$$\therefore f(n) = \frac{2n^3 + 8n^2 + 7n - 2}{n+1}$$

$$f(2) = \frac{16 + 32 + 14 - 2}{3} = \frac{60}{3} = 20$$

Question4

Assertion (A) : $1 + \frac{2 \cdot 1}{3 \cdot 2} + \frac{2 \cdot 5}{3 \cdot 6}\frac{1}{4} + \frac{2 \cdot 5 \cdot 8}{3 \cdot 6 \cdot 9}\frac{1}{8} + \ldots \infty = \sqrt[3]{4}$

Reason (R) : |x| < 1,(1-x) $= 1 + nx + \frac{n(n+1)}{1 \cdot 2}x^2 + \frac{n(n+1)(n+2)}{1 \cdot 2 \cdot 3}x^3 + \ldots$

The correct answer is :

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Options:

A. (A) and (R) are correct, (R) is the correct explanation of (A)

B. (A) and (R) are correct, but (R) is not correct explanalion of (A)

C. (A) is correct but (R) is not correct

D. (A) is not correct but (R) is correct

Answer: A

Solution:

The given series is related to the binomial expansion for any index. The binomial expansion for $(1-x)^{-n}$ when $|x|<1$ is given by :

$(1-x)^{-n}=1+nx+\frac{n(n+1)}{1\cdot2}x^2+\frac{n(n+1)(n+2)}{1\cdot2\cdot3}x^3+\ldots$

To relate the series with the binomial expansion, consider setting $x=\frac{1}{2}$ and $n=\frac{2}{3}$. The substitution into the binomial formula results in :

$\left(\frac{1}{2}\right)^{-\frac{2}{3}}=1+\frac{2\cdot1}{3\cdot2}+\frac{2\cdot5\cdot1}{3\cdot6\cdot4}+\ldots$

This series equates to $\sqrt[3]{4}$:

$\sqrt[3]{4}=1+\frac{2\cdot1}{3\cdot2}+\frac{2\cdot5\cdot1}{3\cdot6\cdot4}+\ldots$

Thus, the assertion that this infinite series sums to $\sqrt[3]{4}$ is correct, and it is derived from the application of the binomial series expansion with the appropriate substitutions for x and n.

Question5

Among the following four statements, the statement which is not true, for all $n \in N$ is

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Options:

A. $(2n+7)<(n+3)^2$

B. $1^2+2^2+\ldots\ldots+n^2>\frac{n^3}{3}$

C. $3\cdot5^{2n+1}+2^{3n+1}$ is divisible by 23

D. $2+7+12+\ldots\ldots+(5n-3)=\frac{n(5n-1)}{2}$

Answer: C

Solution:

We have to check each option, From option (a), we have,

$(2n+7)<(n+3)^2$

Let $P(n) \equiv (2n+7) < (n+3)^2$

$$P(1) = (2+7) = 9 < (1+3)^2 = 4^2 = 16$$

Let us assume that $P(K)$ is true.

We have to prove that $P(K+1)$ is true

$(2K+7) < (K+3)^2$

$2K+7 < K^2+9+6K$

Add 2 on LHS and $7+2K$ on RHS

as $2 < 7+2K$

$\Rightarrow 2K+9 < K^2+16+8K$

$2(K+1)+7 < ((K+1)+3)^2$

$\therefore P(K+1)$ is true.

Hence, $P(x)$ is true, $\forall n \in N$.

From option b,

Let $Q(n) \equiv 1^2+2^2+\ldots+n^2 > \frac{n^3}{3}$

$$Q(1) = 1^2 > \frac{1}{3}$$

Let $Q(K)$ be true.

$1^2+2^2+3^2+\ldots+K^2 > \frac{K^3}{3}$

$1^2+2^2+3^2+\ldots.+K^2$

$+(K+1)^2 > \frac{K^3}{3}+(K+1)^2$

$1^2+2^2+3^2+\ldots.+K^2+(K+1)^2$

$$>\frac{K^3+3K^2+6K+3}{3}$$

$1^2+2^2+3^2+\ldots+K^2+(K+1)^2>$

$$\frac{K^3+3K^2+3K+1+3K+2}{3}$$

$$=\frac{(K+1)^3+3K+2}{3}>\frac{(K+1)^3}{3}$$

$\therefore Q(K+1)^2$ is true.

Hence, $Q(n)$ is true, $\forall n \in N$

From option (c),

$R(n)=35^{2n+1}+2^{3n+1}$

$R(1)=3\times 125+2^4=375+16=391$

which is divisible by 23

Let us assume it is true for $n=k, k \in N$

$\therefore S(k)=3\cdot 5^{2k+1}+2^{3k+1}$ is divisible by

$23 \forall k \in N$

Let $3\cdot 5^{2k+1}+2^{3k+1}=23t, t \in N$

$\therefore 3\cdot 5k^{2k+1}=23t-2^{3k+1}$

Now, we have to prove that for $n=k+1$

$\therefore S(k+1)=3\cdot 5^{2(k+1)+1}+2^{3(k+1)+1}$

$$=3\cdot 5^{2k+1}\cdot 5^2+2^{3k+1}\cdot 2^3$$

$$=\left(23t-2^{3k+1}\right)25+8\cdot 2^{3k+1}$$

$$=23t\times 25-25\times 2^{3k+1}+8\cdot 2^{3k+1}$$

$\therefore S(k+1)=23t\times 25-172^{3k+1}$

Hence, $3\cdot 5^{2n+1}+2^{3n+1}$ is not divisible by 23.

Hence, $R(n)$ is not divisible by $23, \forall n \in N$. From option (d),

$2+7+12+\ldots+(5n-3)$ is an AP with common difference as 5 .

$$S_n=\frac{n}{2}[2\times 2+(n-1)5]$$

$$=\frac{n}{2}(5n-1)$$

Question6

$\frac{1}{3\cdot6}+\frac{1}{6\cdot9}+\frac{1}{9\cdot12}+\ldots\ldots..$ to 9 terms $=$

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Options:

A. $\frac{10}{99}$

B. $\frac{11}{108}$

C. $\frac{1}{10}$

D. $\frac{1}{90}$

Answer: C

Solution:

$$\begin{aligned}
&\frac{1}{3\cdot6}+\frac{1}{6\cdot9}+\frac{1}{9\cdot12}+\ldots \text{ to 9 terms}\\
&=\frac{1}{9}\left(\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\ldots+\frac{1}{9\cdot10}\right)\\
&=\frac{1}{9}\left(\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{9}-\frac{1}{10}\right)\right)\\
&=\left(1-\frac{1}{10}\right)=\frac{1}{9}\left(\frac{9}{10}\right)=\frac{1}{10}
\end{aligned}$$

Question7

When $|x|<2$, then coefficient of x^2 in the power series expansion of $\frac{x}{(x-2)(x-3)}$, is

Options:

A. $\frac{1}{6}$

B. $\frac{5}{36}$

C. $\frac{25}{216}$

D. $\frac{5}{18}$

Answer: B

Solution:

We have,

$\frac{x}{(x-2)(x-3)}=\frac{A}{(x-2)}+\frac{B}{(x-3)}$

On comparing, we get

$A=-2B=3$

$$\Rightarrow \quad \frac{x}{(x-2)(x-3)}=\frac{-2}{x-2}+\frac{3}{x-3}$$

$$=\frac{1}{\left(1-\frac{x}{2}\right)}+\frac{-1}{\left(1-\frac{x}{3}\right)}$$

$$=\left(1-\frac{x}{2}\right)^{-1}-\left(1-\frac{x}{3}\right)^{-1}$$

$$=\left[1+\frac{x}{2}+\left(\frac{x}{2}\right)^{2}+\left(\frac{x}{2}\right)^{3}+\ldots.+\left(\frac{x}{2}\right)^{n}\right]-$$

$$\left[1+\frac{x}{3}+\left(\frac{x}{3}\right)^{2}+\ldots.+\left(\frac{x}{3}\right)^{n}\right]$$

$\Rightarrow$ Coefficient of $x^2=\frac{1}{4}-\frac{1}{9}=\frac{5}{36}$

Question8

The roots of the equation $x^3-14x^2+56x-64=0$ are in

Options:

A. arithmetic-geometric progression

B. harmonic progression

C. arithmetic progression

D. geometric progression

Answer: D

Solution:

$x^3 - 14x^2 + 56x - 64 = 0$

Using Hit and Trial, we can say $x = 2$ is one of the root

$$(x-2)\left(x^2-12x+32\right)=0$$
$$x=2, x^2-12x+32=0$$
$\Rightarrow$ $(x-4)(x-8)=0$

$\Rightarrow$ $x=4$ or $x=8$

$\Rightarrow$ 2,4,8 are roots.

$\Rightarrow$ Roots are in GP.
